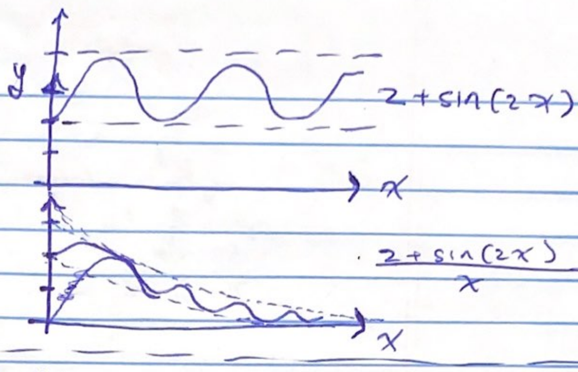
$$\int_1^\infty \frac{2+\sin(2x)}{x}\,dx$$



Limit comparison FAILS

try comparing to $\int_1^\infty \frac{1}{x}\,dx$. $L = \lim_{x\to\infty} \left(\frac{2+\sin(2x)}{x}\right) \Big/ \left(\frac{1}{x}\right)$

$L = \lim_{x\to\infty} (2+\sin(2x))$ limit does not exist

Only choice is direct comparison

observe $1 \le 2+\sin(2x) \le 3$, for all x

thus $\frac{1}{x} \le \frac{2+\sin(2x)}{x}$

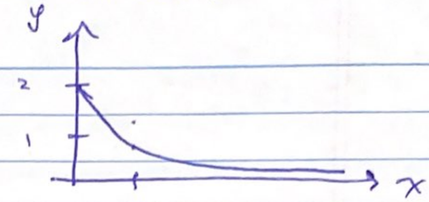
and we have a test function which bounds the integrand from below

and $\int_1^\infty \frac{1}{x}\,dx$ diverges

the direct comparison test is satisfied and the original integral diverges.

Example from (3) Exam 2 October 2019

$$\int_0^{\infty} \frac{1+e^{-x}}{1+x^2}\,dx$$



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•• attempt Limit comparison :

compare with $\int_0^{\infty} \frac{1}{1+x^2}\,dx = \arctan(x)\Big|_{x=0}^{\infty} = \frac{\pi}{2} < \infty$

take $L = \lim_{x\to\infty} \left(\frac{1+e^{-x}}{1+x^2}\right) \Big/ \left(\frac{1}{1+x^2}\right) = \lim_{x\to\infty} (1+e^{-x}) = 1$ non zero non infinite

test is satisfied. original integral converges.

- -

•• attempt Direct comparison :

observe that $1+e^{-x} \leq 2$ for all $x \in [0,\infty)$

then $\frac{1+e^{-x}}{1+x^2} \leq \frac{2}{1+x^2}$ we have a test function

which bounds the integrand from above

and $\int_0^{\infty} \frac{2}{1+x^2}\,dx = 2\arctan(x)\Big|_{x=0}^{\infty} = \pi < \infty$

converges. The test is satisfied

and the original integral converges.

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